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Evaluation of Flight Equipment Acceleration Caused by Random Vibration Loads

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The current approach to the evaluation during the preliminary design phase of random vibration loads acting on flight equipment is based on the use of Miles's equation, assuming the component as a single-degree-of-freedom system subjected to white noise excitation; usually this technique provides too many conservative results with respect to actual accelerations recorded during the flight. In contrast, an accurate evaluation of equipment acceleration can be obtained by running a complete random vibration analysis through a finite element program, e.g., NASTRAN, but because this approch is extremely time comsuming it cannot be applied during the design cycle. The aim here is to describe an accurate and cost-efficient procedure to evaluate the acceleration of components installed on aerospace structures subjected to random vibrations. Several numerical examples and comparisons with results obtained with NASTRAN complete the work.

Nomenclature

$\ddot{a}(\omega)$	=	input acceleration
$\{F_r\}$	=	reaction forces
$\{f\}$	=	excitation for the coupled structure
f_c, ω_c	=	hard-mounted natural frequency of the component
$[K]$	=	stiffness matrix of the secondary structure
k_c	=	stiffness of the connection between component and secondary structure
L_j	=	modal participation factor of the coupled structure
$[l]$	=	modal participation factors of the secondary structure
$[M]$	=	secondary structure mass matrix
$[M_c]$	=	mass matrix related to the component
$[\bar{M}_c]$	=	$[M_c]$ partitioned
$M_{\mathrm{eff},1}$	=	effective mass of the dominant mode
m_c	=	mass of the component
N	=	number of modes of the secondary structure
Q	=	quality factor
Q_{eff}	=	effective amplification factor
$Q_{\mathrm{eff,TC}}$	=	Q_{eff} for the tuning condition between component and secondary structure
$\{q\}$	=	generalized coordinate of elastic modes
$\{q_r\}$	=	translational displacement of the interface
r_j	=	generalized coordinate of the coupled structure
$S(\omega)$	=	input power spectral density
$S_{yc}(f)$	=	acceleration power spectral density of the connection point
$\{V_c\}$	=	mass vector related to the component
$\{\bar{V}_c\}$	=	$\{V_c\}$ partitioned
$\{y\}$	=	degrees of freedom (DOFs) of the secondary structure
y_c	=	displacement of the connection point
$\{y_i\}$	=	internal DOFs of the secondary structure
$\{y_r\}$	=	grounded DOFs of the secondary structure
δ	=	relative displacement of the component, $\delta_a - y_c$
δ_a	=	absolute displacement of the component
ζ	=	damping ratio
$[\Lambda]$	=	eigenvalue matrix (diagonal) of the secondary structure
μ	=	ratio between the component and the secondary structure mass
σ_{yc}	=	standard deviation of the acceleration of the connection point between component and secondary structure
σ_δ	=	standard deviation of the absolute acceleration of the component
$[\Phi]$	=	mode shapes of the secondary structure
$[\Phi_{\mathrm{rr}}]$, $[\Phi_{\mathrm{ir}}]$	=	translational matrices
$\Phi_{yc,1}$	=	mode shape of the dominant mode for the secondary structure, evaluated at the connection point with the flight equipment
$[\Psi]$	=	mode shapes of the coupled structure
ω_j^2	=	eigenvalues of the coupled structure

Subscripts and Superscripts

c	=	component
i	=	ith internal DOF of the secondary structure
j	=	number of eigenmodes of the coupled system
r	=	rth grounded DOF of the secondary structure

I. Introduction

THE structural design of payload, their components, and their supporting structures requires the estimation of various types of loads that act upon them during service. There are three basic types of flight environments that generate dynamic loads on payload components[1]: 1) low-frequency dynamic response of the launch vehicle and payload to transient flight events, 2) high-frequency random vibration environment transmitted from the launch vehicle to the payload, and 3) high-frequency acoustic pressure environment inside the cargo bay.

During both liftoff and the ascent flight, the launch-vehicle thrust produces loads that are transmitted through the structure. Moreover acoustic excitation is produced both by the engine exhaust, when the vehicle is still in proximity of the launchpad, and by aerodynamic disturbances. These loads give rise to random vibrations that are transmitted to the primary structure of the payload and, through the supporting structures (or secondary structures, e.g., mounting frames), to the flight equipment (e.g., electronic boxes, batteries, scientific instruments).

The vibration level of components must be determined in order to ensure the integrity of the entire system. During the preliminary design, engineers use a random spectrum (derived from the design random vibration environment enveloping the maximum input spectra for every secondary structure) as excitation to the supporting structure in order to calculate its dynamic response at the location of the component in terms of power spectral density $S_{yc}(f)$. Moreover, they use this information in order to determine envelope curves that bound the acceleration for the components, depending mainly on

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the component weight as shown by the so-called "mass acceleration curve."[2]

During the preliminary design phase, the component is represented as a single-degree-of-freedom (DOF) system with a known mass, and random vibration loads are taken into account by using equivalent quasi-static loads or load factors.[3] According to Ref. 1, at this stage of the design process the current approach to evaluate the component acceleration is to use the Miles equation[4] assuming the component as a single-DOF system subjected to white noise excitation:

$$\ddot{\delta}_a = \sqrt{\pi f_c S_{yc}(f_c) Q/2} \qquad (1)$$

where $S_{yc}(f)$ is the power spectral density of the acceleration at the component-secondary structure interface and f_c is the natural frequency of the component when its base is assumed to be fixed (i.e., component in hard-mounted condition). Q is the dynamic amplification factor, and for most components the value 10 should be used if no test data are available for all three directions.[3]

Because of the unmodeled dynamic coupling between the secondary structure and the component, the acceleration provided by Miles's equation is higher than the corresponding operational acceleration, giving an excessive conservatism in the designed components and brackets. In general, there is a need to reduce such margins and develop more accurate procedures arriving at load estimates that are adequately but not excessively conservative[5]; indeed, when the predictive random loads are too high, they lead to difficult design problems.[6] As a consequence, an alternative approach is required in order to determine a more accurate estimation of operational random loads acting on flight equipment.

An accurate estimate for loads can be obtained by running a random vibration analysis through the finite element method (FEM). In particular, Thampi and Vidyasagar in Ref. 6 demonstrated, by analyzing a specific subsystem for the Spacelab, that Miles's equation gives an estimate of the acceleration higher than the correct value obtained by running a random vibration analysis through NASTRAN. Even if the use of NASTRAN can solve the problem of determining an accurate estimate for the component acceleration, it is necessary to underline that this calculation requires an extensive computation time. As a result, this approach is not feasible during the design cycle, and more simple techniques are needed.

In Ref. 5 Mehta et al. addressed this problem by analyzing a typical component of a payload and its support structure modeling the coupled structure as a two-DOF system and comparing results obtained both using the Miles equation and solving the coupled equations. Their study demonstrated that results provided by Miles's equation are excessively conservative in a number of cases and in particular when the system damping and/or modal masses increase and when the system has a multimode participation.

A similar problem was addressed by Ceresetti,[7] who proposed a very efficient procedure to determine the component acceleration when the secondary structure has one dominant mode. By analyzing a two-DOF system, Ceresetti demonstrated that the trend of the mass acceleration curve, determined usually by collecting flight data, has a mathematical justification. Moreover, he introduced the so-called "effective amplification factor":

$$Q_{\text{eff}} = \sigma_\delta / \sigma_{yc}$$

to determine the acceleration of the secondary structure as a function of a number of parameters (its hard-mounted natural frequency, damping ratio, and mass ratio μ).

Random vibration testing is usually recommended to verify function of electronic equipment, close tolerance moving devices and optical equipment,[3] and must be performed by exciting the flight equipment at its base with a power spectral density function as close as possible to that actually present during the flight. As a consequence, when direct measurements are not available it is necessary to predict this input spectrum based on a known forcing environment and structure's characteristics. Moreover, as shown in Ref. 8 by Scharton, it is necessary to take into account the dynamic coupling between the flight equipment and its supporting structure to avoid overtesting. As a result, a method allowing the quick estimation of the acceleration at the mounting location of the component is desired for vibration testing too.

The aim of this paper is twofold: first, to propose an approach computationally less involved with respect to the use of a direct FEM solution for random vibration analysis and, second, to take into consideration the main properties of the secondary structure (e.g., the presence of several important modes). In the next section this approach is briefly introduced and then the flight equipment acceleration derived. In Sec. III it is applied to determine the acceleration of a component connected to a truss structure, and corresponding results are compared with those obtained by using NASTRAN.

II. Flight Equipment Acceleration Evaluation

A. Overview of the Proposed Method

The method proposed in this paper permits us to determine an extremely accurate estimate for the acceleration of a flight equipment connected to a secondary structure, as shown in Fig. 1. It is assumed that the secondary structure is connected to the primary structure so that the motion of the latter becomes the excitation for the former. Figure 2 shows that only rigid translations of the interface between secondary and primary structure are considered in this study so that the dynamic response of the secondary structure is obtained by superimposing its rigid-body motion, as a result of the movement of the interface, to its elastic oscillations with respect to the interface.

In the next section it is demonstrated that this approach requires few properties of the secondary structure grounded at the interface with the primary structure: natural frequencies, the corresponding mass-normalized mode shapes (evaluated at the connection point of the considered flight equipment), and the modal participation factors caused by base excitation.

Provided that the modal properties of the grounded secondary structure are already known, the evaluation of the flight equipment acceleration is performed in few steps. It follows that the dynamic properties of the grounded secondary structure can be evaluated

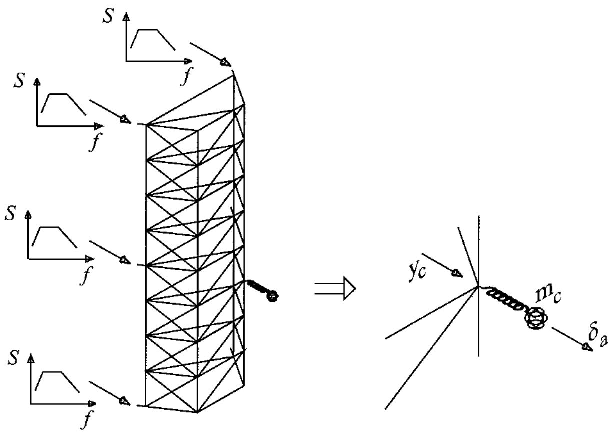

Fig. 1 Flight equipment connected to the supporting structure and the input random vibration.

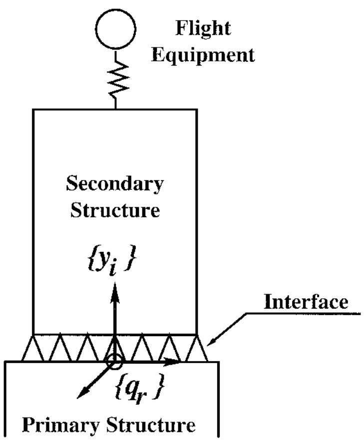

Fig. 2 Translational motion transmitted to the secondary structure.

only once at the beginning of the procedure and can be used several times to couple to this secondary structure a family of flight equipments, e.g., at different locations, with different mass, and/or different rigidity of the connection.

The high computational efficiency of this technique allows a parametric analysis by sweeping the hard-mounted natural frequency of the component. As a result, because the obtained acceleration depends on this parameter as well as on the mounting location on the supporting structure this method permits optimization runs aimed at mass saving. Finally, at the end of the next section it is shown that the power spectral density of the acceleration at the base of the equipment can be easily derived so that engineers involved in component vibration testing can take advantage of this procedure too.

B. Derivation of the Component Acceleration

Figure 1 shows a component of mass m_c connected to a secondary structure. It is assumed that a FE model of the latter structure is available so that $[K]$ and $[M]$ are its stiffness and mass matrices, respectively. By denoting with $\{y\}$ the DOFs of the secondary structure, its potential energy can be written as

$$U_s = \tfrac{1}{2}\{y\}^T[K]\{y\} \tag{2}$$

and its kinetic energy is

$$T_s = \tfrac{1}{2}\{\dot{y}\}^T[M]\{\dot{y}\} \tag{3}$$

Because this procedure is aimed at the preliminary evaluation of the acceleration of the flight equipment, it is assumed that its mass m_c is connected to only one DOF of the secondary structure, denoted with y_c, through a spring with stiffness k_c. As a result, by calling δ the relative displacement of the component with respect to the secondary structure the potential energy of deformation related to the spring is

$$U_c = \tfrac{1}{2}k_c\delta^2 \tag{4}$$

and the kinetic energy of the component is given by

$$T_c = \tfrac{1}{2}m_c(\dot{\delta} + \dot{y}_c)^2 \tag{5}$$

As a consequence, the total potential and kinetic energies are

$$\begin{aligned} U &= \tfrac{1}{2}\{y\}^T[K]\{y\} + \tfrac{1}{2}k_c\delta^2 \\ T &= \tfrac{1}{2}\{\dot{y}\}^T[M]\{\dot{y}\} + \tfrac{1}{2}m_c(\dot{\delta} + \dot{y}_c)^2 \end{aligned} \tag{6}$$

permitting the determination of the stiffness and mass matrices of the coupled structure. Indeed, by introducing $[M_c]$, which is a null matrix with the exception of the element on the y_cth row and column, and $\{V_c\}$, which is a null vector with the exception of the y_cth element, Lagrange's equations can be used to evaluate the mass and stiffness matrices of the complete structure:

$$[M_T] = \begin{bmatrix} [M] + [M_c] & \{V_c\} \\ \{V_c\}^T & m_c \end{bmatrix}, \qquad [K_T] = \begin{bmatrix} [K] & \{0\} \\ \{0\}^T & k_c \end{bmatrix} \tag{7}$$

Usually the secondary structure supporting the component is excited through the connection to the primary structure so that its dynamic response can be calculated by following the same procedure used in Ref. 9 to deal with base-excited systems. As a consequence, the DOFs of the secondary structure can be split into internal DOFs $\{y_i\}$ and grounded DOFs $\{y_r\}$, giving rise to the following equation of motion that highlights the partitioned matrices:

$$\begin{bmatrix} [K_{\rm rr}] & [K_{\rm ri}] & \{0\} \\ [K_{\rm ir}] & [K_{\rm ii}] & \{0\} \\ \{0\}^T & \{0\}^T & k_c \end{bmatrix} \begin{Bmatrix} \{y_r\} \\ \{y_i\} \\ \delta \end{Bmatrix} + \begin{bmatrix} [M_{\rm rr}] & [M_{\rm ri}] & \{0\} \\ [M_{\rm ir}] & [M_{\rm ii}] + [\bar{M}_c] & \{\bar{V}_c\} \\ \{0\}^T & \{\bar{V}_c\}^T & m_c \end{bmatrix} \begin{Bmatrix} \{\ddot{y}_r\} \\ \{\ddot{y}_i\} \\ \ddot{\delta} \end{Bmatrix} = \begin{Bmatrix} \{F_r\} \\ \{0\} \\ 0 \end{Bmatrix} \tag{8}$$

In this equation it is assumed that the component is connected to an internal DOF so that matrix $[M_c]$ and vector $\{V_c\}$ have been partitioned accordingly, giving $[\bar{M}_c]$ and $\{\bar{V}_c\}$, respectively. Moreover, this equation shows that there is not any force acting on both the component and any internal DOF of the secondary structure, i.e., the excitation is transmitted through base motion and only reaction forces $\{F_r\}$ are present.

According to Ref. 9, in the following it is assumed that the interface to the primary structure is whether isostatic or infinitely rigid so that the interface motion can be described by only three DOFs, as shown in Fig. 2: translations along x, y, and z directions (the corresponding three rotations are not considered in this study). As a consequence, the movement of the secondary structure is given by the superposition of the rigid-body motion caused by the interface to the primary structure and the elastic movement with respect to the interface. The last motion is expressed by taking advantage of the elastic modes of the secondary structure fixed at the interface. As a result, the following transformation of variables can be applied:

$$\begin{Bmatrix} \{y_r\} \\ \{y_i\} \\ \delta \end{Bmatrix} = \begin{bmatrix} [\Phi_{\rm rr}] & [0] & \{0\} \\ [\Phi_{\rm ir}] & [\Phi] & \{0\} \\ \{0\}^T & \{0\}^T & 1 \end{bmatrix} \begin{Bmatrix} \{q_r\} \\ \{q\} \\ \delta \end{Bmatrix} = [T] \cdot \begin{Bmatrix} \{q_r\} \\ \{q\} \\ \delta \end{Bmatrix} \tag{9}$$

where $\{q_r\}$ is a three-element vector collecting the translational displacements of the interface along x, y, and z directions. As a result $[\Phi_{\rm rr}]$ and $[\Phi_{\rm ir}]$ are matrices collecting zero and unity elements according to the DOFs orientation as described in Ref. 9. Furthermore, $[\Phi]$ is the matrix containing the structure mode shapes, and $\{q\}$ are the corresponding generalized coordinates that are solution of the eigenvalue problem related to the secondary structure where all of the DOFs of the interface are eliminated:

$$([K_{\rm ii}] - \Lambda_j[M_{\rm ii}])\{\Phi_j\} = \{0\} \tag{10}$$

and satisfy the following two conditions:

$$[\Phi]^T[K_{\rm ii}][\Phi] = [\Lambda], \qquad [\Phi]^T[M_{\rm ii}][\Phi] = [I] \tag{11}$$

with $[I]$ the identity matrix and $[\Lambda]$ a diagonal matrix containing the eigenvalues of the constrained secondary structure.

To determine the dynamic response of the whole structure to base excitation, it is possible to introduce Eq. (9) into Eq. (8) and multiply the latter equation times $[T]^T$ so that three equations are derived.[10] If the base excitation $\{q_r\}$ is known, the second and the third of these equations can be used to evaluate the dynamic response of both the secondary structure and of the component, while the first is used to calculate the reaction forces to the structure motion.

A compact expression can be obtained by recalling properties (11) and that any rigid displacement applied to a structure does not produce any reaction force, so that

$$[K_{\rm rr}][\Phi_{\rm rr}] + [K_{\rm ri}][\Phi_{\rm ir}] = [0], \qquad [K_{\rm ir}][\Phi_{\rm rr}] + [K_{\rm ii}][\Phi_{\rm ir}] = [0]$$

According to these simplifications and considering the base excitation as the input to the whole structure, the second and third equations of the system become

$$\begin{bmatrix} [\Lambda] & \{0\} \\ \{0\}^T & k_c \end{bmatrix} \begin{Bmatrix} \{q\} \\ \delta \end{Bmatrix} + \begin{bmatrix} [I] + [\Phi]^T[\bar{M}_c][\Phi] & [\Phi]^T\{\bar{V}_c\} \\ \{\bar{V}_c\}^T[\Phi] & m_c \end{bmatrix} \begin{Bmatrix} \{\ddot{q}\} \\ \ddot{\delta} \end{Bmatrix} = -\begin{bmatrix} [I] + [\Phi]^T[\bar{M}_c][\Phi_{\rm ir}] \\ \{\bar{V}_c\}^T[\Phi_{\rm ir}] \end{bmatrix} \{\ddot{q}_r\} \tag{12}$$

where the modal participation factors matrix $[l]$ related to base excitation of only the secondary structure is defined as

$$[l] = [\Phi]^T([M_{\mathrm{ii}}][\Phi_{\mathrm{ir}}] + [M_{\mathrm{ir}}][\Phi_{\mathrm{rr}}])$$

To determine the dynamic response of the whole structure, it is possible to solve the forced equation (12) by using modal superposition. As a consequence, first the homogeneous system of equations in (12) is solved to determine the eigenvalues ω_j^2 and corresponding mass-normalized eigenvectors $\{\Psi_j\}$ of the complete structure. By introducing the generalized coordinates $\{r\}$ of the complete structure (i.e., the component connected to the secondary structure constrained at the interface) given by

$$\begin{Bmatrix} \{q\} \\ \delta \end{Bmatrix} = [\Psi]\{r\} \tag{13}$$

into Eq. (12) and multiplying the resulting equation times $[\Psi]^T$, each generalized coordinate r_j can be evaluated taking advantage of the orthogonality of mode shapes with respect to both the mass and stiffness matrix of the complete system:

$$r_j = \frac{\{\Psi_j\}^T\{f\}}{-\omega^2 + \omega_j^2 + 2i\zeta\omega\omega_j} \tag{14}$$

where

$$\{f\} = -\begin{bmatrix} [l] + [\Phi]^T[\bar{M}_c][\Phi_{\mathrm{ir}}] \\ \{\bar{V}_c\}^T[\Phi_{\mathrm{ir}}] \end{bmatrix}\{\ddot{q}_r\} \tag{15}$$

and a viscous damping ratio ζ has been introduced to represent as accurately as possible the dynamic response of real structures.

Effective masses[11–14] play a central role in the evaluation of the dynamic response of structures excited by ground motion, allowing the determination of the most excited elastic modes. With the effective masses matrix of the secondary structure given by $[l]^T[l]$ and matrix $[l]$ present in expression (15), this information can be used in this procedure too in order to select the most important modes of the secondary structure that must be considered during the acceleration evaluation, as shown in the next section.

Usually the excitation to the secondary structure is defined in terms of a single shape in frequency $\ddot{a}(\omega)$ so that the structure motion along x, y, and z directions contained in $\{q_r\}$ can be rewritten as

$$\{\ddot{q}_r\} = \begin{Bmatrix} R_x \\ R_y \\ R_z \end{Bmatrix}\ddot{a}(\omega) \tag{16}$$

As a consequence, the modal participation factor of the coupled structure is

$$\{\Psi_j\}^T\{f\} = L_j\ddot{a} \tag{17}$$

The dynamic response of the component to motions of the secondary structure can be determined by summing the contribution of the modes of the complete structure:

$$\delta(\omega) = \sum_{j=1}^{N+1}\Psi_{N+1,j}\cdot r_j = \sum_{j=1}^{N+1}\left(\frac{\Psi_{N+1,j}L_j}{-\omega^2 + \omega_j^2 + 2i\zeta\omega\omega_j}\right)\ddot{a}(\omega) \tag{18}$$

where it has been assumed that the secondary structure has been represented through N modes and that $\Psi_{N+1,j}$ is the element of the jth mode shape of the complete structure in correspondence of the component.

To determine the equipment acceleration, it can be assumed that it is connected to the secondary structure through a spring and a dashpot. As a result, the equation of motion of the component is

$$(k_c + ic\omega)\delta + m_c\ddot{\delta}_a = 0$$

where c is the damping. This equation allows us to determine the absolute acceleration when the relative displacement is known:

$$\ddot{\delta}_a = -(k_c + ic\omega)/m_c\,\delta = -\left(\omega_c^2 + 2i\zeta\omega\omega_c\right)\delta \tag{19}$$

where ω_c is the natural circular frequency of the hard-mounted component and, because of the use of the proposed procedure during the preliminary design phase, the damping of the component is written in terms of the viscous damping ratio ζ.

By introducing Eq. (18) into Eq. (19), the absolute acceleration of the component can be evaluated as

$$\ddot{\delta}_a(\omega) = \sum_{j=1}^{N+1}\left(-\Psi_{N+1,j}L_j\frac{\omega_c^2 + 2i\zeta\omega\omega_c}{-\omega^2 + \omega_j^2 + 2i\zeta\omega\omega_j}\right)\ddot{a}(\omega) \tag{20}$$

When the base motion is a random excitation, it is known in terms of power spectral density defined as[15,16]

$$S(\omega) = \lim_{T\to\infty}\frac{|\ddot{a}(\omega)|^2}{2\pi T} \tag{21}$$

where T is the duration of the process. Moreover, because the excitation is random in nature only the standard deviation σ_δ of the response is important for the engineer: assuming a Gaussian probability density function for this random process, the maximum value of the acceleration is usually considered as $3\sigma_\delta$ (Ref. 3) or derived as shown in Ref. 17. The variance σ_δ^2 of the absolute acceleration of the component can be evaluated by recalling that[15,16]

$$\sigma_\delta^2 = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\delta_a^2(t)\,\mathrm{d}t = \lim_{T\to\infty}\int_{-\infty}^{\infty}\frac{|\ddot{\delta}_a(\omega)|^2}{2\pi T}\,\mathrm{d}\omega$$
$$= \lim_{T\to\infty}2\int_0^{\infty}\frac{|\ddot{\delta}_a(\omega)|^2}{2\pi T}\,\mathrm{d}\omega \tag{22}$$

and, by introducing expression (20) into (22),

$$\sigma_\delta^2 = 2\int_0^{\infty}\left|\sum_{j=1}^{N+1}\left(-\Psi_{N+1,j}L_j\frac{\omega_c^2 + 2i\zeta\omega\omega_c}{-\omega^2 + \omega_j^2 + 2i\zeta\omega\omega_j}\right)\right|^2 S(\omega)\,\mathrm{d}\omega \tag{23}$$

This relation clearly shows that the acceleration of the component depends on several parameters:

1) The damping ratio ζ is the parameter that is usually derived during experimental vibration tests, and, according to Ref. 3, when experimental data are not available can be fixed to 0.05, a typical value for this kind of structures. It should be recalled that for lower values of ζ the coupling terms arising in expression (23) as a result of the square can be neglected, permitting to approximate the result by a closed formula, as shown in Sec. 5.2 of Ref. 16. In contrast, for values as high as 0.05 the result of the integral can be evaluated only through numerical integration.

2) The natural frequency of the hard-mounted component ω_c is the parameter that affects heavily the dynamic response to random excitation; indeed, as demonstrated by the theory on vibration absorbers, if ω_c is close to a natural frequency of the secondary structure, the acceleration of the component will be highest.

3) The mass of the component m_c is the parameter that is present in Eq. (12), affecting the value of both natural frequencies and mode shapes of the complete structure.

4) The power spectral density of the excitation $S(\omega)$ is the function that is usually limited in a given range of frequencies so that the integral extends over a limited range.[5]

To summarize, in order to use Eq. (23) to determine the acceleration of the component it is necessary to apply the following procedure:

1) Run a normal mode analysis on the FE model of the secondary structure without the component in order to determine a) its natural frequencies, b) corresponding modal participation factors $l_{j,k}$ for base excitation along k direction, and c) mode shapes at the mounting point of the equipment.

2) Fix the directions of the excitation, i.e., set values for R_x, R_y, and R_z.

3) Select the modes of the secondary structure having effective masses greater than 1% of the structure total mass for the selected directions.

4) Fix the component mass m_c and its hard-mounted natural frequency f_c.

5) Evaluate eigenvalues and eigenvectors of the homogeneous equation (12).

6) Calculate the modal participation factors L_j for the coupled structure.

7) Determine the variance of the absolute acceleration of the equipment by integrating numerically relation (23).

Finally, as stated in the introduction of this article, the proposed procedure allows us to determine easily the acceleration power spectral density at the connection point between the equipment and the supporting structure, extremely useful information for vibration testing. Indeed, the equation of motion of the equipment can be rewritten as

$$(k_c + ic\omega)(\delta_a - y_c) + m_c\ddot{\delta}_a = 0$$

permitting us to determine the relation between the acceleration of the component and the acceleration at its base:

$$\ddot{y}_c = \frac{-\omega^2 + \omega_c^2 + 2i\zeta\omega\omega_c}{\omega_c^2 + 2i\zeta\omega\omega_c}\ddot{\delta}_a \tag{24}$$

By introducing expression (20) into (24), the acceleration at the base can be rewritten as

$$\ddot{y}_c(\omega) = \sum_{j=1}^{N+1}\left(-\Psi_{N+1,j}L_j\frac{-\omega^2 + \omega_c^2 + 2i\zeta\omega\omega_c}{-\omega^2 + \omega_j^2 + 2i\zeta\omega\omega_j}\right)\ddot{a}(\omega) \tag{25}$$

and, similarly to the procedure permitting the determination of the standard deviation of the dynamic response of the component, the power spectral density of the acceleration at the mounting location of the flight equipment is

$$S_{yc}(\omega) = \left|\sum_{j=1}^{N+1}\left(-\Psi_{N+1,j}L_j\frac{-\omega^2 + \omega_c^2 + 2i\zeta\omega\omega_c}{-\omega^2 + \omega_j^2 + 2i\zeta\omega\omega_j}\right)\right|^2 S(\omega) \tag{26}$$

This last equation shows how to modify the input power spectral density $S(\omega)$ to take into account both the dynamic properties of the secondary structure and its dynamic coupling with the flight equipment. In particular, expression (26) confirms that at the hard-mounted natural frequency of the component the acceleration of the connection point is low.

III. Numerical Example

The proposed method has been used to evaluate the acceleration of simulated components on the simple truss represented in Fig. 3 and considered as a secondary structure. Its overall length is 6 m, the height 1 m, every member is built up in aluminum with Young's modulus of 7×10^{10} N/m^2, density of 2700 kg/m^2, and cross-sectional area of 1×10^{-3} m^2, so that its total mass is 71.5 kg. A quality factor Q of 10 is used; moreover, the constrained secondary structure has 24 DOFs, whereas, as shown in the following, its dynamic response caused by base excitation will be determined by using, at maximum, eight modes. (Even if in a realistic structure the number of DOFs present in the FE model is extremely large, usually few modes are necessary to calculate the acceleration of the flight equipment, as shown in Ref. 10 in the analysis of some secondary structures of node number 2 of the International Space Station.)

Natural frequencies and corresponding effective masses of the first nine modes are listed in Table 1, showing that only one mode, at 144.4 Hz, is dominant for an excitation along the x direction. Conversely, most important modes for base excitation along the y direction have natural frequencies of 22.5 and 91.9 Hz.

As a result, even if this supporting structure is very simple, when it is excited along the y direction it is not possible to apply a method

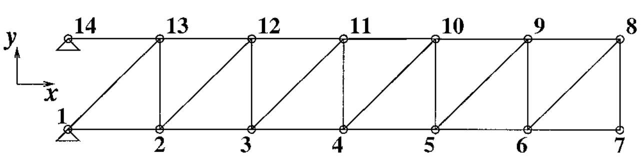

Fig. 3 Supporting structure under analysis.

Table 1 Natural frequencies and corresponding effective masses for the secondary structure under analysis

Mode no.	Natural frequency, Hz	Effective mass direction X, %	Effective mass direction Y, %
1	22.5	0.08	64.61
2	91.9	2.73	20.16
3	144.4	77.39	0.63
4	191.6	0.02	4.95
5	277.3	0.12	1.94
6	346.6	0.51	0.52
7	391.7	2.15	0.43
8	406.2	3.61	0.01
9	458.1	2.46	0.04

Table 2 Position and direction of the component on the supporting structure, direction of the excitation

Case study	Node	Direction excitation	Direction component	Considered modes
A	8	x	x	2,3,7,8,9
B	8	y	y	1,2,4,5
C	8	y	x	1–5, 7–9
D	11	x	x	2,3,7,8,9
E	11	y	y	1,2,4,5
F	11	y	x	1–5, 7–9

dealing with a two-DOF system (one describing the secondary structure and the other representing the equipment), as in Refs. 5 and 7: in this case a more refined technique is needed.

A. Parametric Analysis

Six case studies have been analyzed, A to F, according to Table 2 where modes of the secondary structure involved in the calculation are shown. In every case it has been assumed that the component is connected to a node of the supporting structure through a simple spring, and its motion is permitted along one direction only. Furthermore, in cases A to C the component is connected at node 8, whereas in cases D to F it is connected to node 11 so that the influence of the equipment location can be investigated. Finally, in cases A, B, D, and E excitation and response directions are coincident, whereas in cases C and F they are different.

To simulate the vibration transmission from the primary to the secondary structure, the latter has been subjected to a base excitation with power spectral density $S(f)$ equal to 0.02 g^2/Hz in the range from 1 to 500 Hz. Moreover, it has been assumed that vibrations are transmitted to the secondary structure along whether direction x or direction y.

To validate the coupled analysis procedure described in the preceding section, some comparisons have been performed with results obtained by running a complete random vibration analysis with NASTRAN. The standard deviation of the absolute acceleration of the equipment is listed in Tables 3 and 4 as a function of the hard-mounted natural frequency of the equipment and of the ratio μ between the component and the secondary structure mass. The corresponding values shown in Tables 3 and 4 are in close agreement; the small differences are caused by the limited number of modes of the supporting structure utilized by the proposed procedure. Every run has a duration of about 30 s for the complete random vibration analysis on a workstation, whereas just 2 s are necessary to determine the acceleration when the procedure proposed in the preceding section is applied and the corresponding program is run on a personal computer.

Figures 4–9 show the standard deviation of the absolute acceleration $\ddot{\delta}_a$ of the equipment for the six case studies as a function of both the mass ratio μ and the hard-mounted natural frequency of the component.

Figures 4 and 7 illustrate the acceleration for both base excitation and component orientation along the x direction and show that the maximum dynamic response of the component is in correspondence of the tuning condition between mode 3, with maximum effective mass for the x direction, and the hard-mounted natural frequency of the equipment.

Table 3 Comparison between component acceleration evaluated by NASTRAN and by the present method (component at node 8)

			σ_δ, g	
Case study	f_c, Hz	μ	NASTRAN	Present method
A	91.9	0.001	11.40	11.46
	91.9	0.01	10.87	10.98
	144.4	0.001	55.43	55.73
	144.4	0.01	34.66	34.96
	391.7	0.001	21.10	20.92
	391.7	0.01	16.99	17.33
	406.2	0.001	21.64	21.61
	406.2	0.01	17.15	17.47
	458.1	0.001	22.72	23.22
	458.1	0.01	15.96	15.98
B	22.5	0.001	23.94	23.93
	22.5	0.01	13.39	13.40
	91.9	0.001	22.09	22.19
	91.9	0.01	14.24	14.29
	144.4	0.001	9.08	9.07
	144.4	0.01	8.52	8.51
	191.6	0.001	14.65	14.69
	191.6	0.01	11.60	11.62
	277.3	0.001	9.69	9.78
	277.3	0.01	8.11	8.04

Table 4 Comparison between component acceleration evaluated by NASTRAN and by the present method (component at node 11)

			σ_δ, g	
Case study	f_c, Hz	μ	NASTRAN	Present method
D	91.9	0.001	8.64	8.68
	91.9	0.01	8.56	8.61
	144.4	0.001	34.09	34.24
	144.4	0.01	27.26	27.39
	391.7	0.001	23.98	24.12
	391.7	0.01	15.58	16.14
	406.2	0.001	25.30	25.09
	406.2	0.01	16.06	16.40
	458.1	0.001	19.27	18.61
	458.1	0.01	14.54	14.12
E	22.5	0.001	10.58	10.55
	22.5	0.01	8.94	8.93
	91.9	0.001	20.23	20.32
	91.9	0.01	13.51	13.57
	144.4	0.001	6.74	6.78
	144.4	0.01	6.44	6.48
	191.6	0.001	6.00	6.09
	191.6	0.01	5.80	5.89
	277.3	0.001	12.40	12.51
	277.3	0.01	8.34	8.40

A similar result is shown in Figs. 5 and 8, where both the excitation and the component are orientated along the y direction, even if in these cases there are two maxima in the response, in correspondence of modes number 1 and 2 of the secondary structure, that have the maximum effective masses for y direction. Furthermore, it is interesting to see that for the component located at node 8 the most critical tuning condition is with the mode at 22.5 Hz, whereas for location at node 11 the critical tuning condition is with the mode at 91.9 Hz: a result clearly related to the shape of modes 1 and 2.

Finally, Figs. 6 and 9 refer to case studies in which the direction of excitation and the component orientation are different, a common situation for components mounted on the node 2 of the International Space Station, giving rise, in several cases, to very high accelerations.[10] It must be emphasized that in these cases it is no longer possible to determine few dominant modes according to effective masses listed in Table 1; as a result simple methods as those proposed in Refs. 5 and 7 cannot be used. Conversely, the technique proposed in this article provides as accurate results as those obtained for the other four case studies. In particular, Figs. 6 and 9 show that there is a local maximum in the dynamic response of the equipment at every mode of the secondary structure.

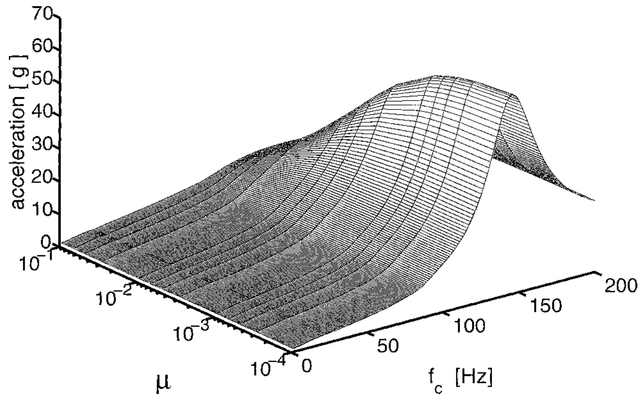

Fig. 4 Component acceleration for case A.

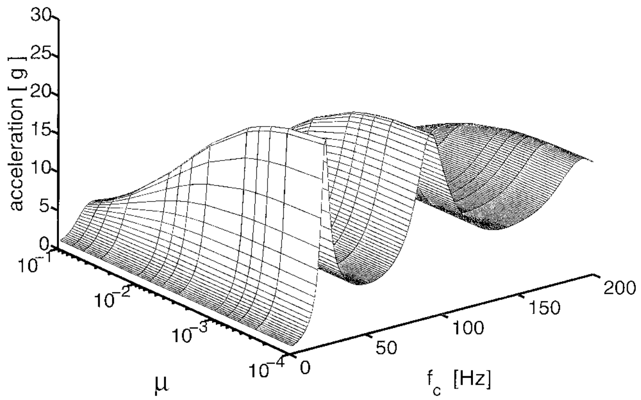

Fig. 5 Component acceleration for case B.

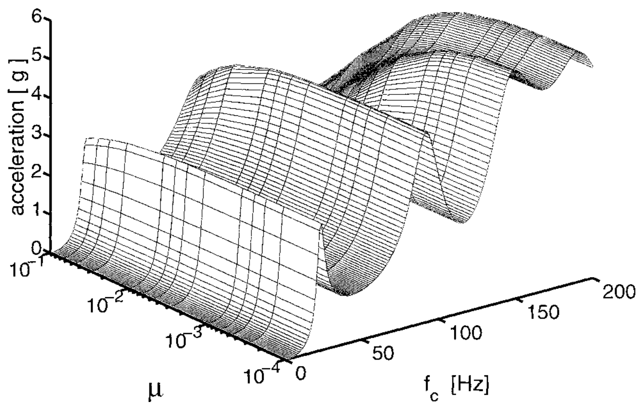

Fig. 6 Component acceleration for case C.

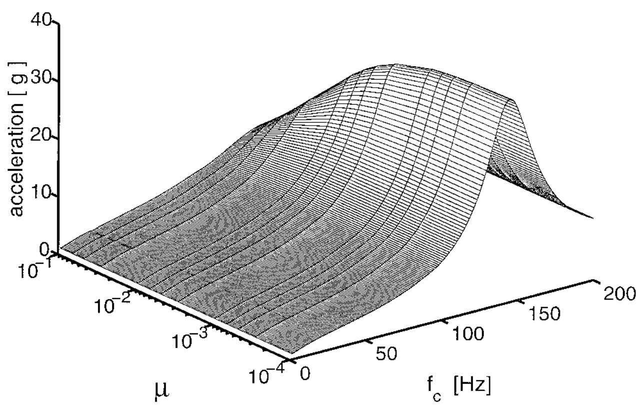

Fig. 7 Component acceleration for case D.

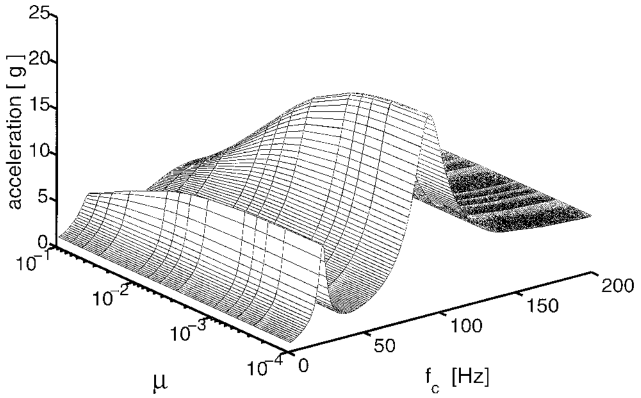

Fig. 8 Component acceleration for case E.

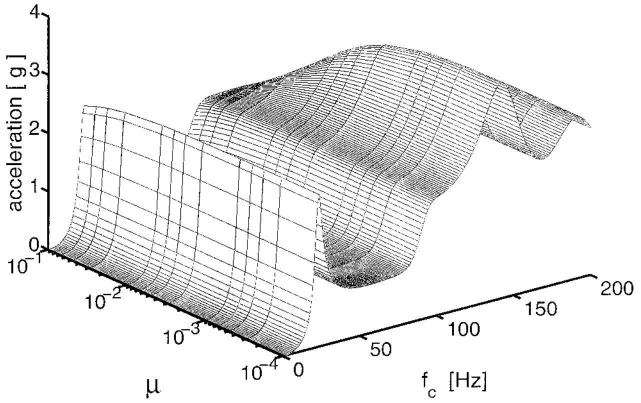

Fig. 9 Component acceleration for case F.

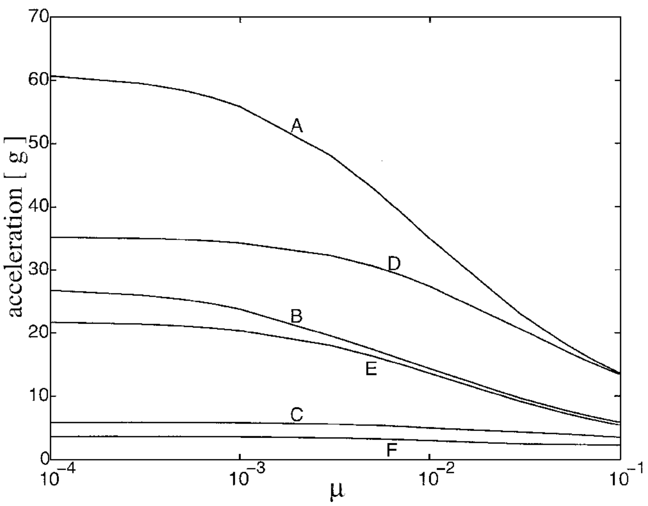

Fig. 10 Comparison of maximum accelerations for the six analyzed case studies.

Results similar to those summarized in Figs. 4 to 9 are extremely useful during the preliminary design phase, allowing us to determine the most critical condition for a component of a given mass ratio μ as a function of its hard-mounted natural frequency, i.e., of the stiffness of the connection with the secondary structure. As a consequence, it is possible to determine the best stiffness of the connection in order to minimize the acceleration of the considered component. From this point of view and recalling the low computation time required by this procedure, it permits optimization runs aimed at the reduction of the dynamic load on the equipment and, as a consequence, to save mass.

B. Envelope of the Acceleration

By plotting an envelope of the maximum dynamic responses of the equipment by sweeping the natural frequency f_c, as shown in Fig. 10 for all of the six case studies, the most critical mounting position on the secondary structure can be located: it is node number 8, with a corresponding maximum standard deviation of the acceleration of about 60 g for a very light equipment. In contrast, acceleration levels in cases C and F, i.e., when excitation and component are oriented along different directions, are very low.

The envelope shown in Fig. 10 is extremely similar to the mass acceleration curve discussed in Ref. 2 and analyzed by Ceresetti in Ref. 7. In the following it is demonstrated that for very light components, i.e., when $\mu \rightarrow 0$, the asymptote of the curve is related to no dynamic loading caused by the component motion on the secondary structure; it follows that, in this case, the dynamic response of the secondary structure can be determined by neglecting the presence of the flight equipment. In contrast, the acceleration of the component is calculated as a function of the acceleration of its connection point and considering its dynamic coupling with the secondary structure.

When the secondary structure has only one dominant mode, the acceleration of the flight equipment can be easily evaluated. In this case the envelope curve has an asymptote whose value can be obtained by applying the Miles equation to the secondary structure and assuming that the component is tuned to the natural frequency of the dominant mode.

In this case the maximum acceleration of the flight equipment is given by the acceleration of its connection point to the secondary structure σ_{yc} times the effective amplification factor under tuning condition[7]:

$$\sigma_\delta = Q_{\mathrm{eff,TC}} \cdot \sigma_{yc}$$

For very light components the standard deviation of the acceleration of the connection point can be calculated by using the Miles equation modified according to the dynamic properties of the structure:

$$\sigma_{yc} = |\Phi_{yc,1}|\sqrt{M_{\mathrm{eff},1}}\sqrt{\pi f_1 S(f_1) Q/2}$$

where $M_{\mathrm{eff},1}$ is the effective mass of the dominant mode for the considered interface motion and $\Phi_{yc,1}$ is its mass-normalized mode shape evaluated at the connection point with the flight equipment. For case A $M_{\mathrm{eff},1} = 55.34$ kg, $\Phi_{yc,1} = -0.1715$, $f_1 = 144.4$ Hz, $S(f_1) = 0.02$ g^2/Hz, and $Q = 10$ so that $\sigma_{yc} = 8.59$ g.

Moreover, in Ref. 7 it is demonstrated that the maximum value for the effective amplification factor under tuning condition is 7.2 so that it follows $\sigma_\delta = 7.2 \cdot \sigma_{yc} = 61.9$ g. Figure 10 shows that this is the value of the asymptote for case A.

To summarize, the preceding calculation demonstrates that for light components and for cases A and D, i.e., when the secondary structure has a single dominant mode, the coupled system behaves as a tuned two DOF. In the other four cases the secondary structure has, at least, two dominant modes so that Miles's equation cannot be used to determine the acceleration of the connection point on the secondary structure. Nevertheless, the physical meaning remains unchanged, i.e., no dynamic loading caused by the component on the secondary structure.

IV. Conclusions

In this article a procedure aimed at evaluating the acceleration of components connected to a secondary structure is proposed. It is useful during the preliminary design phase, in particular when 1) a finite element model of secondary structures is available, 2) only the mass of the component is known, and 3) it is necessary to estimate the maximum in-flight acceleration. This result can be derived through a parametric analysis by sweeping the hard-mounted natural frequency of the component, a task that at the moment is extremely expensive being based on several complete random vibration analysis run with NASTRAN. In constrast, as shown in the preceding section, the proposed procedure allows us to run this parametric study in very limited time, being based on modal superposition and coupled analysis.

It is well known that the application of the Miles equation provides an estimate of the equipment acceleration, which is 30–40%

in excess with respect to the actual value. As a result, the main advantage of the proposed procedure over this classical technique is that it provides an accurate estimate of the acceleration of the equipment, taking into consideration the dynamic behavior of the secondary structure, e.g., the presence of several dominant modes, as well as the mounting location.

As a side result, this technique permits us to determine easily the power spectral density of the acceleration at the connection point between the flight equipment and the supporting structure, very useful information during vibration testing of the component.

By enveloping dynamic response curves, this procedure allows us to determine the equivalent of the mass acceleration curve,[2] i.e., the maximum acceleration as a function of the ratio μ between the mass of the component and of the secondary structure. From this point of view, even if transient loads are not considered, the presented coupled analysis gives a mathematical justification to the mass acceleration curve that, usually, is derived from previous experience and/or recorded flight data as stated in Ref. 1.

Finally, the high computation effeciency of this method opens new possibilities in terms of optimizing the stiffness of the connection between the component and the supporting structure as well as the mounting location, with the final aim of saving mass and reduce costs.

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References

[1]"Loads Analyses of Spacecraft and Payloads," NASA STD-5002, June 1996.

[2]Trubert, M., "Mass Acceleration Curve for Spacecraft Structural Design," Jet Propulsion Lab., California Inst. of Technology, JPL D-5882, Pasadena, CA, Nov. 1989.

[3]"ISS Payload Flight Equipment Requirements and Guidelines for Safety of Critical Structures," NASA International Space Station Program, SSP 52005, Johnson Space Center, Houston, TX, Nov. 1995.

[4]Miles, J. W., "On Structural Fatigue Under Random Loading," *Journal of the Aeronautical Sciences*, Vol. 21, No. 11, 1954, pp. 753–762.

[5]Mehta, R., Erwin, E., Suryanarayan, S., and Krishna, M. M. R., "Improved Estimation of Random Vibration Loads in Launch Vehicles," AIAA Paper 93-1092, Feb. 1993.

[6]Thampi, S. K., and Vidyasagar, S. N., "Random Vibration Analysis of Space Flight Hardware Using NASTRAN," *COSMIC, 18th NASTRAN Users' Colloquium*, 1990, pp. 90–106.

[7]Ceresetti, A., "Limit Load Factors Curve (L.L.F.) for Preliminary Mechanical Design of Components for Space Missions—An Attempt of Mathematical Justification," *European Conference on Spacecraft Structures, Materials and Mechanical Testing*, ESA, SP-428, Noordwjik, The Netherlands, 1998, pp. 179–188.

[8]Scharton, T. D., "Force Limited Vibration Testing Monograph," NASA RP-1403, July 1997.

[9]Imbert, J. F., *Analyse des Structures par Elements Finis*, 2nd ed., Cepadues Editions, Toulouse, France, 1984, pp. 414–417.

[10]Cotterchio, M., "Acceleration of Equipments Subjected to Random Vibrations," M.S. Thesis, Dept. of Aeronautical and Space Engineering, Politecnico di Torino, Turin, Italy, May 2000 (in Italian).

[11]Imbert, J. F., *Analyse des Structures par Elements Finis*, 2nd ed., Cepadues Editions, Toulouse, France, 1984, pp. 426–438.

[12]Girard, A., "Modal Effective Mass Models in Structural Dynamics," *IX International Modal Analysis Conference*, April 1991, pp. 45–50.

[13]Géradin, M., and Rixen, D., *Mechanical Vibrations—Theory and Application to Structural Dynamics*, Wiley, New York, 1994, Chap. 2.

[14]Wada, B. K., Bamford, R. M., and Garba, J. A., "Equivalent Spring-Mass System: a Physical Interpretation," *Shock and Vibration Bullettin*, No. 42, Pt. 5, 1972, pp. 215–225.

[15]Crandall, S. H., *Random Vibration*, Vol. 2, MIT Press, Cambridge, MA, 1963, pp. 1–12.

[16]Yang, C. Y., *Random Vibration of Structures*, Wiley, New York, 1986, Chap. 5.

[17]Sarafin, T. P., and Larson, W. J., *Spacecraft Structures and Mechanisms—From Concept to Launch*, Kluwer Academic, Norwell, MA, 1995, Sec. 2.2.

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